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3D measurements from a single uncalibrated image based on estimating space planes using geometrical clues in real scenes

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Abstract
In this paper, we propose four new plane estimation methods using only single uncalibrated image. It is well known that only one image cannot provide enough 3D information, but some metrical quantities can be estimated directly from the single image by using geometrical clues such as planarity of points and parallelism of lines and planes. Specifically in this paper, we utilize four new geometrical clues to estimate a specified space plane; the first is known length on the plane, the second is known angle on the plane, the third is true circle and its center position on the plane, and the last is two circles on the plane or its parallel ones. Furthermore, we estimate several 3D entities such as a distance between two space points from the single image by using estimated planes. Experiments using some real images verified that our methods give good 3D information.

1. Introduction

It is important research to extract and reconstruct 3D structures in real world from 2D images. For this research, the stereo method is used, which inputs two or more images taken from different viewpoints (multi-view images). However, the stereo method has a problem that it is difficult to match corresponding points accurately between images and that their position error greatly affects estimation accuracy of 3D entities. Moreover, we cannot use the stereo method if we target old image data and images taken by fixed surveillance camera. Thus in this paper, we focus on an issue to extract 3D entities by using a single uncalibrated image, without multi-view images.

If we use a single viewpoint image, we usually need some clues about scenes (for example, several imaged points lie on the same plane in the real world) since we use only one image. As described in references [2] to [5], 3D measurement methods by using various clues have been suggested. Among them, Wang[4] et al. shows that the plane intersecting with a fixed reference plane could be estimated in a unified frame from some clues in the image. Furthermore, Wang et al. verified that precise 3D entities could be extracted from each of the estimated plane parameters and implemented more accurate measurement methods than conventional ones. But their method left some problems such as the image of intersection lines must be visible in the image or a projection matrix must be calculated again if another plane was regarded as the reference plane. Therefore in this paper, we first generalize the plane estimation method in which the reference plane is not fixed. From this, an estimated arbitrary space plane can be a new reference plane, and space planes can be estimated in order without recalculating a projection matrix. We also show that the plane intersecting obliquely with a reference plane can be obtained from four kinds of new clues. It is expected that our method can spread the range of applicable images and the versatile three-dimensional measurement method can be established.

In the following sections, we give the outline of necessary projective geometry and explain the details of our method.

2. Camera model and preliminary knowledge

2.1. Notation

A space point in the world XYZ coordinate system is described as $\boldsymbol{X} = [X, Y, Z]^{\mathrm{T}}$ using the capital letters of alphabet. The point in uv image coordinate system corresponding to the space point is described as $\boldsymbol{m} = [u, v]^{\mathrm{T}}$ using the small letters. The homogeneous vector for space point $\boldsymbol{X}$ is described as $\tilde{\boldsymbol{X}} = [X, Y, Z, 1]^{\mathrm{T}}$ and that for image point $\boldsymbol{m}$ as $\tilde{\boldsymbol{m}} = [u, v, 1]^{\mathrm{T}}$. $(\boldsymbol{a}, \boldsymbol{b})$ represents the inner product of 3D vector $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{a} \times \boldsymbol{b}$ represents the cross product of $\boldsymbol{a}, \boldsymbol{b}$.

2.2. Camera projection matrix

The processing of space point $\boldsymbol{X}$ projected to image point $\boldsymbol{m}$ by a camera is modeled by using 3×4 matrix $P = [p_{ij}]$ as follows.

$$\lambda \tilde{\boldsymbol{m}} = P\tilde{\boldsymbol{X}} = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4]\tilde{\boldsymbol{X}} = K[R, \boldsymbol{t}]\tilde{\boldsymbol{X}}, \quad (1)$$

where λ having depth information of the 3D space. The degree of freedom for projection matrix P is 11 because it can only be defined meaningfully up to a scaling factor. K is called camera intrinsic matrix including the parameters unique for the camera to be used. Matrix $[R, \boldsymbol{t}]$ is called camera extrinsic matrix representing transform between the world coordinate system and the camera coordinate system, where matrix R and vector $\boldsymbol{t}$ represent rotation and translation respectively.

Here, we show a lemma about projection matrix P (for the proof, please refer to [1]).

lemma 1. *For the line $\boldsymbol{l}$ in an image, its back-projection is a plane including camera center $\boldsymbol{o}_c$ and straight line $\boldsymbol{l}$, and the plane can be written as $\boldsymbol{\pi}_b = P^{\mathrm{T}}\boldsymbol{l}$ using projection matrix P.*

2.3. Homography

If a base plane $\boldsymbol{\pi}_0 = [0, 0, 1, 0]^{\mathrm{T}}$ in space is supposed to be the reference plane (the world coordinate system is determined so as to be $Z = 0$), we have the equation below from Eq.(1) about space point $\boldsymbol{X} = [X, Y, 0, 1]^{\mathrm{T}}$ on the reference plane:

$$\begin{aligned} \lambda \tilde{\boldsymbol{m}} &= P[X, Y, 0, 1]^{\mathrm{T}} \\ &= [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_4][X, Y, 1]^{\mathrm{T}} = H\tilde{\boldsymbol{x}}_r. \end{aligned} \quad (2)$$

This means that 2D point $\tilde{\boldsymbol{x}}_r = [X, Y, 1]^{\mathrm{T}}$ on the reference plane is projected to point $\tilde{\boldsymbol{m}}$ on the image plane by 3×3 regular matrix H, which is called homography. The degree of freedom is 8 (up to scale). If correspondence for four pairs of points between the reference and image planes is determined, H can be calculated.

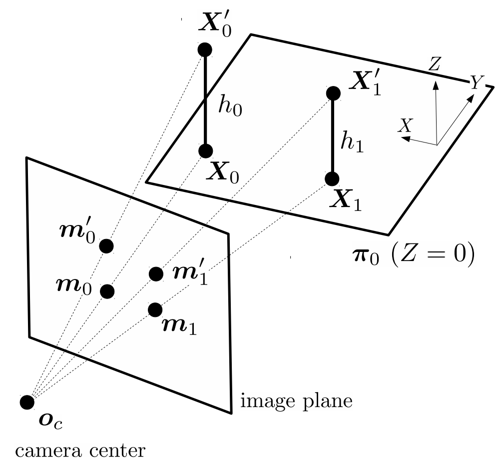

Figure 1: Estimation of projection matrix P.

2.4. Conic

A curve described by a second-degree equation in the plane is called a conic, which can be represented by the following equation:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0.$$

If $\tilde{\boldsymbol{x}} = [x, y, 1]^{\mathrm{T}}$ is provided, this equation can be expressed by the following quadratic form:

$$(\tilde{\boldsymbol{x}}, C\tilde{\boldsymbol{x}}) = \tilde{\boldsymbol{x}}^{\mathrm{T}} C \tilde{\boldsymbol{x}} = 0,$$

where

$$C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}.$$

In the following, C is used as the matrix representing a conic.

Specifically, the absolute conic Ω_∞ is a conic on the plane at infinity $\boldsymbol{\Pi}_\infty = [0, 0, 0, 1]^{\mathrm{T}}$, which can be expressed as $\Omega_\infty = I_3 = \mathrm{diag}(1, 1, 1)$. Ω_∞ is composed of purely imaginary points on the $\boldsymbol{\Pi}_\infty$.

Here, some lemmas are shown about conics (for the proof, please refer to [1]).

lemma 2. *The relationship between pole $\boldsymbol{x}$ and polar $\boldsymbol{l}$ about conic C is expressed as $\boldsymbol{l} = C\tilde{\boldsymbol{x}}$. In particular, when C is a true circle, the polar of circle center $\boldsymbol{x}_c$ is the line at infinity $\boldsymbol{L}_\infty$ ($\boldsymbol{L}_\infty = C\tilde{\boldsymbol{x}}_c$).*

lemma 3. *Arbitrary circle Ω_c in a space plane always intersects with absolute conic Ω_∞ at two points. These two points I, J ($= [1, \pm i, 0]^{\mathrm{T}}$) are called circular points of the plane.*

lemma 4. *Image ω of absolute conic Ω_∞ is expressed by $\omega = (KK^{\mathrm{T}})^{-1}$ using only camera intrinsic matrix K.*

3. Estimation of projection matrix

The key to 3D measurements is estimation of projection matrix P. If camera is calibrated, we can define $P = K[R, \boldsymbol{t}]$. But this paper handles uncalibrated images, it is necessary to estimate P with some kind of clues about the scene. Although references [2] and [4] describe the concrete estimation methods, P is estimated based on the following proposition in this paper.

proposition 1. *If homography H about base plane $\boldsymbol{\pi}_0 = [0, 0, 1, 0]^{\mathrm{T}}$ is known, projection matrix P can be estimated by two imaged points of two end points where heights from $\boldsymbol{\pi}_0$ are both known.*

Proof. As Fig.1 shows, suppose that two end points of a segment where height from $\boldsymbol{\pi}_0$ is h_0 are described as $\tilde{\boldsymbol{X}}_0 = [X_0, Y_0, 0, 1]^{\mathrm{T}}$ and $\tilde{\boldsymbol{X}}_0' = [X_0, Y_0, h_0, 1]^{\mathrm{T}}$, and that of another segment where height is h_1 are $\tilde{\boldsymbol{X}}_1 = [X_0, Y_0, 0, 1]^{\mathrm{T}}$ and $\tilde{\boldsymbol{X}}_1' = [X_0, Y_0, h_1, 1]^{\mathrm{T}}$. Also, suppose four image points corresponding to $\tilde{\boldsymbol{X}}_i$, $\tilde{\boldsymbol{X}}_i'$ ($i = 0, 1$) are described as $\tilde{\boldsymbol{m}}_i = [u_i, v_i, 1]^{\mathrm{T}}$, $\tilde{\boldsymbol{m}}_i' = [u_i', v_i', 1]$ ($i = 0, 1$), respectively. The homography $H = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_4]$ is known, so here it is needed to estimate only the third vector $\boldsymbol{p}_3$ of P.

The X, Y coordinates of $\boldsymbol{X}_0$ and $\boldsymbol{X}_1$ on $\boldsymbol{\pi}_0$ can be calculated from Eq.(2) as

$$[X_i, Y_i, 1]^{\mathrm{T}} = \lambda_i H^{-1}\tilde{\boldsymbol{m}}_i \quad (i = 0, 1). \tag{3}$$

Note that λ_i can be removed by the third equation of Eq.(3). For space points $\boldsymbol{X}'_0$ and $\boldsymbol{X}'_1$,

$$\begin{cases} \lambda_0\tilde{\boldsymbol{m}}'_0 = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4]\tilde{\boldsymbol{X}}'_0 \\ \lambda_1\tilde{\boldsymbol{m}}'_1 = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4]\tilde{\boldsymbol{X}}'_1 \end{cases} \tag{4}$$

is obtained from Eq.(1). Since λ_0, λ_1 can be removed from the above equation,

$$A\boldsymbol{p}_3 = \boldsymbol{b} \tag{5}$$

is obtained, while

$$A = \begin{bmatrix} 0 & 0 & -u'_0h_0 \\ 0 & 0 & -v'_0h_0 \\ h_1 & 0 & -u'_1h_1 \\ 0 & h_1 & -v'_1h_1 \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} (p_{31}u'_0-p_{11})X_0+(p_{32}u'_0-p_{12})Y_0+(p_{34}u'_0-p_{14}) \\ (p_{31}v'_0-p_{21})X_0+(p_{32}v'_0-p_{22})Y_0+(p_{34}v'_0-p_{24}) \\ (p_{31}u'_1-p_{11})X_1+(p_{32}u'_1-p_{12})Y_1+(p_{34}u'_1-p_{14}) \\ (p_{31}v'_1-p_{21})X_1+(p_{32}v'_1-p_{22})Y_1+(p_{34}v'_1-p_{24}) \end{bmatrix}. \tag{6}$$

Therefore, $\boldsymbol{p}_3$ is given as the following equation.

$$\boldsymbol{p}_3 = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}\boldsymbol{b} \tag{7}$$

□

In this paper, correspondence of four points to define homography H is given manually and RQ decomposition[1] is used to obtain $K, R, \boldsymbol{t}$ from the estimated projection matrix P.

The estimation method based on proposition 1 is a similar to the Tsai[7] method, so-called *6 points algorithm*. In that point of view, our method has an advantage that there is no need to determine an accurate vanishing point of orthogonal direction for calculating projection matrix, compared with other estimation methods such as Wang[4] et al. method.

4. Estimation of plane

When projection matrix P is determined, planes in 3D space can be estimated only from an image. This paper describes further generalization of the Wang[4] et al. method and suggests a method to estimate a plane from four new clues.

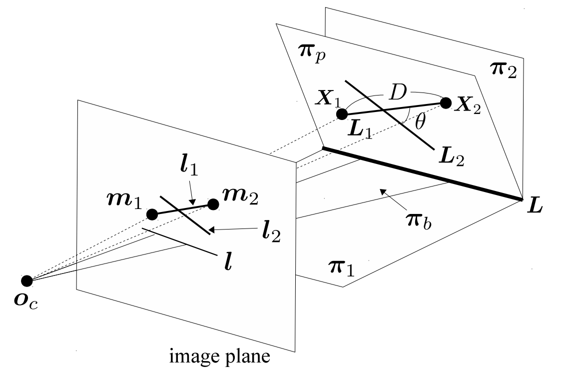

Figure 2: Two kinds of clues: known length D and known angle θ on $\boldsymbol{\pi}_p$ ($\boldsymbol{\pi}_1$ is reference plane, $\boldsymbol{\pi}_2$ is vertical plane, and $\boldsymbol{\pi}_p$ is arbitrary plane).

4.1. Estimation of a plane perpendicular to a reference plane

Suppose a reference plane $\boldsymbol{\pi}_1 = [a_1, b_1, c_1, d_1]^{\mathrm{T}}$ is known. For a plane $\boldsymbol{\pi}_2 = [a_2, b_2, c_2, d_2]^{\mathrm{T}}$ intersecting the plane $\boldsymbol{\pi}_1$ perpendicularly with the line $\boldsymbol{L}$, we have the following proposition (see also Fig.2).

proposition 2. *Plane $\boldsymbol{\pi}_2 = [a_2, b_2, c_2, d_2]^{\mathrm{T}}$ intersecting the reference plane $\boldsymbol{\pi}_1$ perpendicularly with the line $\boldsymbol{L}$ can be determined by using back-projection $P^{\mathrm{T}}\boldsymbol{l}$ of intersection line image $\boldsymbol{l}$.*

Proof. From lemma 1, a plane as the back-projection of intersection line image $\boldsymbol{l}$ is given as $\boldsymbol{\pi}_b = P^{\mathrm{T}}\boldsymbol{l}$. Since three planes $\boldsymbol{\pi}_1, \boldsymbol{\pi}_2$, and $\boldsymbol{\pi}_b$ form the pencil in which intersection line $\boldsymbol{L}$ is shared, $\boldsymbol{\pi}_2$ is expressed by using unknown scalar ν as

$$\boldsymbol{\pi}_b = \nu\boldsymbol{\pi}_1 + \boldsymbol{\pi}_2. \tag{8}$$

If we put $\boldsymbol{\pi}_b = [a_3, b_3, c_3, d_3]^{\mathrm{T}}$, the following equation is obtained from Eq.(8) since plane $\boldsymbol{\pi}_1$ intersects with $\boldsymbol{\pi}_2$ perpendicularly,

$$\nu = \frac{a_1a_3 + b_1b_3 + c_1c_3}{a_1^2 + b_1^2 + c_1^2}. \tag{9}$$

Therefore $\boldsymbol{\pi}_2$ is given as the following equation.

$$\boldsymbol{\pi}_2 = P^{\mathrm{T}}\boldsymbol{l} - \nu\boldsymbol{\pi}_1. \tag{10}$$

□

Proposition 2 is corresponding to a generalization of *proposition 3* in [4] shown by Wang et al. Since their method assumes a fixed reference plane $[0, 0, 1, 0]^{\mathrm{T}}$, a plane adjacent to the reference plane can only be estimated. In other words, if we want to estimate *the next plane* adjacent to the first estimated plane by using Wang et al. method, projection matrix P must be recalculated so as to the first estimated plane is set to $[0, 0, 1, 0]^{\mathrm{T}}$ as a new reference plane on a new world coordinate system. On the other hand, our method assumes a generic reference plane $[a_1, b_1, c_1, d_1]^{\mathrm{T}}$. Thus the base plane $\boldsymbol{\pi}_0$ can be used as the first reference plane, and then a newly estimated space plane will be the next reference plane without recalculating projection matrix P [1].

4.2. Estimation of a plane intersecting at an arbitrary angle with a reference plane

As Fig.2 shows, plane $\boldsymbol{\pi}_p$ that intersects with line $\boldsymbol{L}$ at an arbitrary angle can be written as the pencil that shares intersection line L between reference plane $\boldsymbol{\pi}_1 = [a_1, b_1, c_1, d_1]^{\mathrm{T}}$ and vertical plane $\boldsymbol{\pi}_2 = [a_2, b_2, c_2, d_2]^{\mathrm{T}}$ as follows:

$$\boldsymbol{\pi}_p = \mu\boldsymbol{\pi}_1 + \boldsymbol{\pi}_2. \tag{11}$$

This paper proposes a method to obtain unknown scalar μ from four new clues described below.

4.2.1. Method with a known length on $\boldsymbol{\pi}_p$

As Fig.2 shows, suppose that two points $\boldsymbol{X}_1, \boldsymbol{X}_2$ exist on plane $\boldsymbol{\pi}_p$ and the real length D from $\boldsymbol{X}_1$ to $\boldsymbol{X}_2$ is already known. Let the imaged points of $\tilde{\boldsymbol{X}}_1, \tilde{\boldsymbol{X}}_2$ be $\tilde{\boldsymbol{m}}_1 = [u_1, v_1, 1]^{\mathrm{T}}, \tilde{\boldsymbol{m}}_2 = [u_2, v_2, 1]^{\mathrm{T}}$, respectively.

proposition 3. *Plane $\boldsymbol{\pi}_p$ can be calculated by using the $\tilde{\boldsymbol{m}}_1, \tilde{\boldsymbol{m}}_2$. However, both of $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ do not exist on the intersection line L.*

[1] Thus, an implementation program can probably be written by simpler code.

Proof. By Eq.(1) and $\boldsymbol{X}_1, \boldsymbol{X}_2 \in \boldsymbol{\pi}_p$, the following equations hold for 3D points $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$:

$$\begin{cases} \lambda_i\tilde{\boldsymbol{m}}_i = P\tilde{\boldsymbol{X}}_i \\ \boldsymbol{\pi}_p^{\mathrm{T}}\tilde{\boldsymbol{X}}_i = 0 \end{cases} \quad (i = 1, 2).$$

Since λ_i can be removed from the former equation,

$$\Sigma_i\boldsymbol{X}_i = \boldsymbol{\gamma}_i, \ (i = 1, 2) \tag{12}$$

is obtained, while

$$\Sigma_i = \begin{bmatrix} p_{11} - p_{31}u_i & p_{12} - p_{32}u_i & p_{13} - p_{33}u_i \\ p_{21} - p_{31}v_i & p_{22} - p_{32}v_i & p_{23} - p_{33}v_i \\ a_2 + \mu a_1 & b_2 + \mu b_1 & c_2 + \mu c_1 \end{bmatrix},$$

$$\boldsymbol{\gamma}_i = \begin{bmatrix} p_{34}u_i - p_{14} \\ p_{34}v_i - p_{24} \\ -d_2 - \mu d_1 \end{bmatrix} \quad (i = 1, 2).$$

Therefore, $\boldsymbol{X}_i$ is given as:

$$\boldsymbol{X}_i = \frac{1}{(\alpha)_i\mu + (\beta)_i}\begin{bmatrix} (f_1)_i\mu + (g_1)_i \\ (f_2)_i\mu + (g_2)_i \\ (f_3)_i\mu + (g_3)_i \end{bmatrix} \ (i = 1, 2), \tag{13}$$

where the coefficients $(\alpha)_i$, $(\beta)_i$, $(f_1)_i$, $(g_1)_i$, $(f_2)_i$, $(g_2)_i$, $(f_3)_i$ and $(g_3)_i$ are all scalar values and their concrete expressions are given in Appendix 4.2.1. Since the real length from $\boldsymbol{X}_1$ to $\boldsymbol{X}_2$ is D, the following equation holds:

$$\|\boldsymbol{X}_2 - \boldsymbol{X}_1\|^2 = D^2. \tag{14}$$

Thus, by substituting Eq.(13) to Eq.(14), the following quartic equation of μ is obtained:

$$k_4\mu^4 + k_3\mu^3 + k_2\mu^2 + k_1\mu + k_0 = 0. \tag{15}$$

The concrete expressions of coefficients k_4, k_3, k_2, k_1, k_0 are given in Appendix 4.2.1. Solve this quartic equation of μ and select a proper one from real solutions, plane $\boldsymbol{\pi}_p$ can be determined. □

4.2.2. Method with a known angle on $\boldsymbol{\pi}_p$

As Fig.2 shows, suppose that two lines $\boldsymbol{L}_1, \boldsymbol{L}_2$ exist on plane $\boldsymbol{\pi}_p$, and the angle θ formed by them is already known.

proposition 4. *Plane $\boldsymbol{\pi}_p$ can be calculated by using two imaged lines $\boldsymbol{l}_1, \boldsymbol{l}_2$ of $\boldsymbol{L}_1, \boldsymbol{L}_2$ if the intersecting angle θ on $\boldsymbol{\pi}_p$ is already known.*

Proof. From lemma1, the back-projected planes $\boldsymbol{\pi}_{b1}, \boldsymbol{\pi}_{b2}$ of imaged lines $\boldsymbol{l}_1, \boldsymbol{l}_2$ are given as:

$$\boldsymbol{\pi}_{bi} = P^{\mathrm{T}}\boldsymbol{l}_i \ (i = 1, 2).$$

If we put $P = [M, \boldsymbol{p}_4]$ ($M = [\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3]$), the normal vectors $\boldsymbol{n}_{b1}, \boldsymbol{n}_{b2}$ of planes $\boldsymbol{\pi}_{b1}, \boldsymbol{\pi}_{b2}$ are given as:

$$\boldsymbol{n}_{bi} = M^{\mathrm{T}}\boldsymbol{l}_i \ (i = 1, 2).$$

Meanwhile, from Eq.(11), the normal vector $\boldsymbol{n}_p$ of plane $\boldsymbol{\pi}_p$ is expressed by using normal vectors $\boldsymbol{n}_1, \boldsymbol{n}_2$ of planes $\boldsymbol{\pi}_1, \boldsymbol{\pi}_2$ vertical with each other as:

$$\boldsymbol{n}_p = \mu\boldsymbol{n}_1 + \boldsymbol{n}_2. \tag{16}$$

Since the line $\boldsymbol{L}_i$ $(i = 1, 2)$ on plane $\boldsymbol{\pi}_p$ is the intersecting line of planes $\boldsymbol{\pi}_p$ and $\boldsymbol{\pi}_{bi}$, the directional vector $\boldsymbol{d}_i$ of $\boldsymbol{L}_i$ is orthogonal to both of $\boldsymbol{n}_p, \boldsymbol{n}_{bi}$ and given as:

$$\begin{aligned}\boldsymbol{d}_i &= \boldsymbol{n}_p \times \boldsymbol{n}_{bi} \\ &= \mu(\boldsymbol{n}_1 \times \boldsymbol{n}_{bi}) + (\boldsymbol{n}_2 \times \boldsymbol{n}_{bi}) \ (i = 1, 2).\end{aligned} \tag{17}$$

Since the angle formed by $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ is θ, the following equation holds:

$$\cos\theta = \frac{\boldsymbol{d}_1^{\mathrm{T}}\boldsymbol{d}_2}{\sqrt{\boldsymbol{d}_1^{\mathrm{T}}\boldsymbol{d}_1}\sqrt{\boldsymbol{d}_2^{\mathrm{T}}\boldsymbol{d}_2}}. \tag{18}$$

Thus, by substituting Eq.(17) to Eq.(18), the following quartic equation of μ is obtained:

$$k_4'\mu^4 + k_3'\mu^3 + k_2'\mu^2 + k_1'\mu + k_0' = 0. \tag{19}$$

The concrete expressions of coefficients $k_4', k_3', k_2', k_1', k_0'$ are given in Appendix 4.2.2. Solve this quartic equation of μ and select a proper one from all real solutions, plane $\boldsymbol{\pi}_p$ can be determined.

It should be noted that if the line $\boldsymbol{L}_1$ is orthogonal to the line $\boldsymbol{L}_2$, Eq.(19) will be a quadratic equation of μ because of $\boldsymbol{d}_1^{\mathrm{T}}\boldsymbol{d}_2 = 0$. In case that the line $\boldsymbol{L}_1$ is parallel to the line $\boldsymbol{L}_2$, Eq.(19) will be a linear equation of μ because of $\boldsymbol{d}_1 \times \boldsymbol{d}_2 = \boldsymbol{0}$ and $\boldsymbol{n}_1^{\mathrm{T}}\boldsymbol{n}_2 = 0$. □

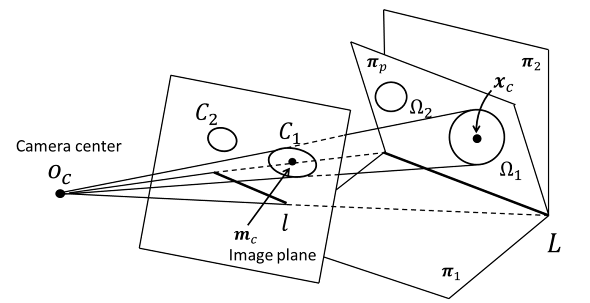

Figure 3: Another two kinds of clues: true circle Ω_1 and its center $\boldsymbol{x}_c$, and two true circles Ω_1, Ω_2 on $\boldsymbol{\pi}_p$ ($\boldsymbol{\pi}_1$ is reference plane, $\boldsymbol{\pi}_2$ is vertical plane, and $\boldsymbol{\pi}_p$ is arbitrary plane).

4.2.3. Method with a true circle and its center on π_p

As Fig.3 shows, suppose that true circle Ω_1 exists on plane $\boldsymbol{\pi}_p$. The image of Ω_1 is assumed to be C_1 and the image corresponding to circle center $\tilde{\boldsymbol{x}}_c$ of Ω_1 to be $\tilde{\boldsymbol{m}}_c$.

proposition 5. *Plane $\boldsymbol{\pi}_p$ can be calculated by using the C_1 and $\tilde{\boldsymbol{m}}_c$.*

Proof. The normal vectors of planes $\boldsymbol{\pi}_p, \boldsymbol{\pi}_1$, and $\boldsymbol{\pi}_2$ are given as $\boldsymbol{n}_p, \boldsymbol{n}_1$, and $\boldsymbol{n}_2$, respectively. From Eq.(11), we have the following normal vector:

$$\boldsymbol{n}_p = \mu\boldsymbol{n}_1 + \boldsymbol{n}_2. \tag{20}$$

From lemma 2, vanishing line $\boldsymbol{l}_\infty$ of plane $\boldsymbol{\pi}_p$ is obtained by the following expression:

$$\boldsymbol{l}_\infty = C_1\tilde{\boldsymbol{m}}_c. \tag{21}$$

Since plane $\boldsymbol{\pi}_b$ as the back-projection of the vanishing line $\boldsymbol{l}_\infty$ should be parallel to plane $\boldsymbol{\pi}_p$, the normal vectors $\boldsymbol{n}_b$ and $\boldsymbol{n}_p$ will also be parallel. From lemma 1, $\boldsymbol{\pi}_b = P^{\mathrm{T}}\boldsymbol{l}_\infty$ is given, the normal vector of plane $\boldsymbol{\pi}_b$ is:

$$\boldsymbol{n}_b = [(P^{\mathrm{T}}\boldsymbol{l}_\infty)_1, (P^{\mathrm{T}}\boldsymbol{l}_\infty)_2, (P^{\mathrm{T}}\boldsymbol{l}_\infty)_3]. \tag{22}$$

Since $\boldsymbol{n}_p$ is parallel to $\boldsymbol{n}_b$, the following is obtained:

$$\boldsymbol{n}_b \times \boldsymbol{n}_p = \boldsymbol{0}. \tag{23}$$

Thus, Eq.(20) is substituted to Eq.(23) to solve the equation about μ. Then μ is substituted to Eq.(11), plane $\boldsymbol{\pi}_p$ can be determined. □

4.2.4. Method with the image of two true circles on $\boldsymbol{\pi}_p$

As Fig.3 shows, suppose true circles Ω_1, Ω_2 exists on plane $\boldsymbol{\pi}_p$ [2] and the images of each circle are C_1, C_2 respectively.

proposition 6. *Plane $\boldsymbol{\pi}_p$ can be calculated from C_1 and C_2.*

Proof. According to the Bézout's theorem, two conics always intersect at four points[3]. Therefore, each of Ω_1 and Ω_∞, and Ω_2 and Ω_∞ on plane $\boldsymbol{\pi}_p$ contains four intersection points. However, from lemma 3, two points among the four are equal to the circular points $I = [1, i, 0]^{\mathrm{T}}, J = [1, -i, 0]^{\mathrm{T}}$.

Since the relationship described above do not change after projective transform, it is applied also to the image plane. In other words, each of image C_1 and ω, and image C_2 and ω contains four intersection points and the common points should become image $\tilde{\boldsymbol{m}}_I, \tilde{\boldsymbol{m}}_J$ of the circular points I, J. If the following is given:

$$C_1 = \begin{bmatrix} a_1 & h_1/2 & g_1/2 \\ h_1/2 & b_1 & f_1/2 \\ g_1/2 & f_1/2 & 1 \end{bmatrix},$$

$$\omega = (KK^{\mathrm{T}})^{-1} \equiv \begin{bmatrix} \omega_1 & \omega_2/2 & \omega_3/2 \\ \omega_2/2 & \omega_4 & \omega_5/2 \\ \omega_3/2 & \omega_5/2 & 1 \end{bmatrix},$$

the intersection points will be obtained by solving the following:

$$\begin{cases} a_1x^2 + h_1xy + b_1y^2 + g_1x + f_1y + 1 = 0 \\ \omega_1x^2 + \omega_2xy + \omega_4y^2 + \omega_3x + \omega_5y + 1 = 0. \end{cases} \tag{24}$$

In other words, y is removed from Eq.(24) to obtain a quartic equation of x, and four intersection points are calculated. Solve it for C_2 and ω in the same manner and calculate the solution common to the former one, then $\tilde{\boldsymbol{m}}_I$,$\tilde{\boldsymbol{m}}_J$ are determined. Since $\tilde{\boldsymbol{m}}_I$,$\tilde{\boldsymbol{m}}_J$ are imaginary circular points, the line joined with $\tilde{\boldsymbol{m}}_I$ and $\tilde{\boldsymbol{m}}_J$ should be a vanishing line (of plane $\boldsymbol{\pi}_p$). Since vanishing line $\boldsymbol{l}_\infty$ is

$$\boldsymbol{l}_\infty = \tilde{\boldsymbol{m}}_I \times \tilde{\boldsymbol{m}}_J,$$

the subsequent operations must follow the procedure of the latter half of the proof of proposition 5. □

[2] A plane parallel to $\boldsymbol{\pi}_p$ is also available.

[3] Ideal points must be considered in the range of complex number.

5. Specification of 3D point from image point

If projection matrix P and space plane $\boldsymbol{\pi} = [a, b, c, d]^{\mathrm{T}}$ are both estimated, various 3D entities can be estimated from an image. The base proposition is as follows:

proposition 7. *Arbitrary 3D space point $\boldsymbol{X} = [X, Y, Z]^{\mathrm{T}}$ on space plane $\boldsymbol{\pi}$ can be calculated from the corresponding image point $\boldsymbol{m}$ if $\boldsymbol{\pi}$ is already known.*

Proof. By Eq.(1) and $\boldsymbol{X} \in \boldsymbol{\pi}$,

$$\begin{cases} \lambda\tilde{\boldsymbol{m}} = P\tilde{\boldsymbol{X}} \\ \boldsymbol{\pi}^{\mathrm{T}}\boldsymbol{X} = 0 \end{cases} \tag{25}$$

is obtained. If λ is removed from this equation,

$$A\boldsymbol{X} = \boldsymbol{b}$$

is obtained, while

$$A = \begin{bmatrix} p_{11} - p_{31}u & p_{12} - p_{32}u & p_{13} - p_{33}u \\ p_{12} - p_{31}v & p_{22} - p_{32}v & p_{23} - p_{33}v \\ a & b & c \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} p_{34}u - p_{14} \\ p_{34}v - p_{24} \\ -d \end{bmatrix}. \tag{26}$$

This must be solved for $\boldsymbol{X}$. □

If this is used, various quantities can be measured from only a single viewpoint image, such as the distance between two points on a plane, angle between two lines, or distance between space points belonging to different planes.

6. Experiment

To verify validity and effectiveness of our method, we had experiments with two input images (Figs. 4 and 6) photographed by NIKON D7000 camera in our university campus. The resolution of

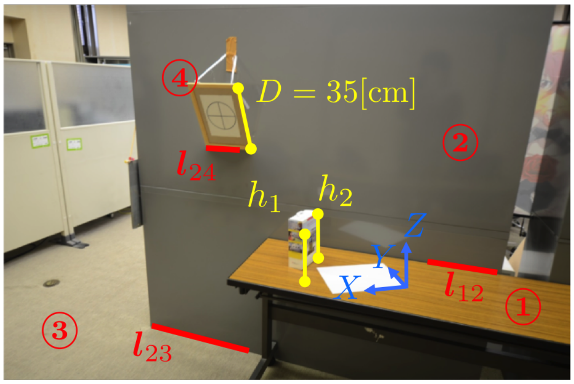

Figure 4: Test image 1.

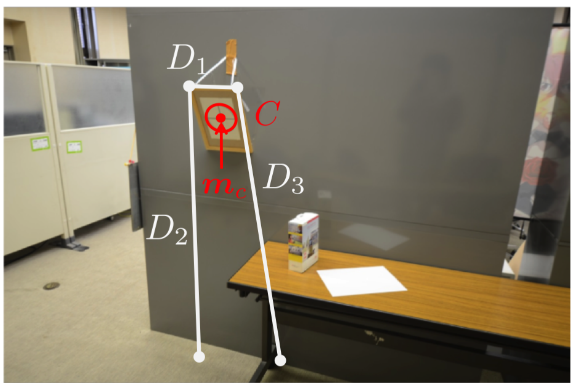

Figure 5: Three spatial distances to be estimated.

each image is 640×424 pixels. Fig.4 is the image where a true circle is drawn in the frame (plane ④) hung obliquely on the back of the gray shelf (plane ②) behind the desk. In this image, the top of the table was set as the first reference plane $\boldsymbol{\pi}_1$ (plane ①) and one of the four corner points of the A4-size paper on the table was set as the origin of the world coordinate system. From the correspondence of the four corner points, homography was calculated. Using reference height $h_1 = h_2 = 21.0$[cm] shown in the figure, the projection matrix P was estimated according to proposition 1. In addition, using the estimated projection matrix, space plane ② was estimated from intersection line image $\boldsymbol{l}_{12}$ and proposition 2, space plane ③ from intersection line image $\boldsymbol{l}_{23}$ and proposition 2. Moreover, space plane ④ was estimated from intersection line image $\boldsymbol{l}_{24}$ and each of two kinds of clues: proposition 3 ($D = 35$[cm] shown in the figure) and proposition 5. In case using proposition 5, ELSD method[6] of V.pătrăucean et al. was used to extract imaged circle (i.e. ellipse) C automatically, and imaged center point $\boldsymbol{m}_c$ was specified manually (see Fig.5). Then, spatial distances D_1, D_2 and D_3 between two points were estimated using the specified space planes ②, ③, ④ and proposition 7. The result was shown in Table 1. The true values in the table were the values obtained by measuring actual objects manually. Note that D_3 indicates the distance measured from the right-upper corner of the frame to the point on the intersecting line of floor and gray shelf. (The lower point locates near the tip of the leg of the table, so the line D_3 breaks through the desk.) The relative errors against the measured values were also placed in the table. From the values in the table, we can confirm that the result is valid. Our experiment showed that dimensions difficult to measure in actual space, such as D_3, could be estimated by using our method.

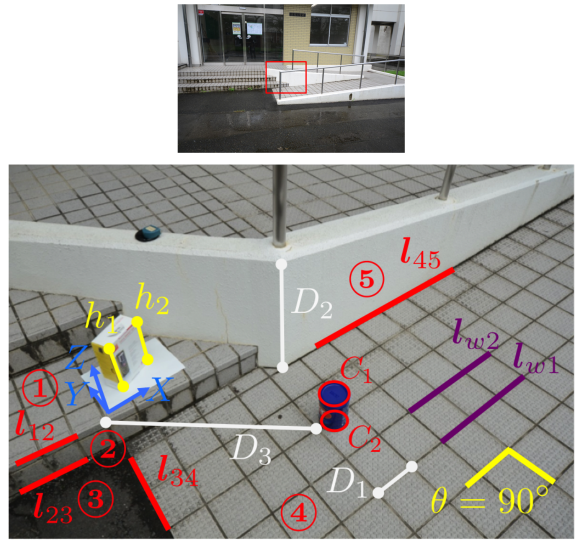

Figure 6: Test image 2.

Subsequently we had another experiment shown in Fig.6 where a part of the building entrance was enlarged. This figure shows the scene where a blue coffee can was placed on the slope (plane ④) intersecting with the ground (plane ③) obliquely. One step of the stairs was set as the first reference plane $\boldsymbol{\pi}_1$ (plane ①) and projection matrix P was estimated from the A4-size paper placed on the step and reference height $h_1 = h_2 = 21.0$[cm]. Using the estimated projection matrix, space plane ② was estimated

Table 1: Estimation result of test image 1

	Ground truth	Estimation results with our method			
		Using Prop.3	Relative error [%]	Using Prop.5	Relative error [%]
D_1 [cm]	24.1	24.2	0.42	24.6	2.0
D_2 [cm]	138.8	142.9	3.0	142.3	2.4
D_3 [cm]	144.5	147.4	2.0	148.6	2.8

Table 2: Estimation result of test image 2

	Ground truth	Estimation results with our method			
		Using Prop.4	Relative error [%]	Using Prop.6	Relative error [%]
D_1 [cm]	14.2	14.1	0.70	14.0	1.4
D_2 [cm]	40.2	40.8	1.5	41.1	2.2
D_3 [cm]	70.1	71.1	1.4	71.3	1.7

from intersection line image $\boldsymbol{l}_{12}$ and proposition 2, ground plane ③ from intersection line image $\boldsymbol{l}_{23}$ and proposition 2. The slope plane ④ was estimated from intersection line image $\boldsymbol{l}_{34}$ and each of two kinds of clues: proposition 4 ($\theta = 90°$ shown in the figure [4]) and proposition 6, and space plane ⑤ was estimated from intersection line image $\boldsymbol{l}_{45}$ and proposition 2. Here, ELSD was used again to extract imaged circles C_1 and C_2 necessary for using proposition 6 The result of estimation of spatial lengths D_1, D_2 and D_3 from the image is shown in Table 2. Also in this experiment, the errors were not so large and the valid result was obtained.

Note that there are many parallel lines in space plane ④ of Fig.6. Thus in this case, Wang[4] et al. method is also available to estimate plane ④. For example, using a pair of $\boldsymbol{l}_{w1}$ and $\boldsymbol{l}_{w2}$, the vanishing point for the plane ④ could be determined. Using this, lengths D_1 and D_3 could be calculated as 13.6, 68.7[cm], and the relative errors are 4.2, 2.0[%], respectively. In their method, estimation accuracy is a little bit lower than our method. This is caused the plane ④ was not properly estimated since the calculated position of the vanishing point was far from the center of the image. In a similar case, known angle or two circle clue of our method may also be successful.

[4]Note that we have only to solve the quadratic equation since $\theta = 90°$.

7. Conclusion

In this paper, we described the method to estimate a space plane using some clues appeared in a single uncalibrated image and to extract 3D information according to the clues. In particular, we showed that an arbitrary plane inclining against a reference plane could be estimated from four new clues: a known length on the plane, a known angle on the plane, a true circle and its center point on the plane, and two true circles on the plane or its parallel ones. From the evaluation experiment with real images, the effectiveness of our method could be confirmed, while we found an issue that in the method using proposition 3, 4, or 6, where significant digits are canceled in the process of solving quartic equations and the precision of solution is lowered. There is another issue that the performance of our method greatly depends on detection precision for lines and ellipses in an image. Moreover, it is necessary to examine in detail which clue is superior and effective for what kind of case using more general or actual examples. Also it is need to study and discuss the merits and demerits comparing our method with such as Wang[4] et al. method. We will study these issues further in the future.

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A. Coefficients derivation of quartic equation in Section 4.2.1

In Eq.(12), each component of matrix Σ_i and vector $\boldsymbol{\gamma}_i$ is expressed as:

$$\Sigma_i = [(\sigma_{jk})_i],\ \boldsymbol{\gamma}_i = [(\gamma_l)_i]\ (j,k,l=1,2,3).$$

The determinant of submatrix consisted of eliminating j row and k column from Σ_i is expressed as $|\Sigma_{jk}|_i$, each coefficient of Eq.(13) is obtained by using $\boldsymbol{X}_i = \Sigma_i^{-1}\boldsymbol{\gamma}_i$:

$$\begin{aligned}
(\alpha)_i &= a_1|\Sigma_{31}|_i - b_1|\Sigma_{32}|_i + c_1|\Sigma_{33}|_i \\
(\beta)_i &= a_2|\Sigma_{31}|_i - b_2|\Sigma_{32}|_i + c_2|\Sigma_{33}|_i \\
(f_1)_i &= (\alpha_{11})_i(\gamma_1)_i - (\alpha_{21})_i(\gamma_2)_i - |\Sigma_{31}|_i d_1 \\
(g_1)_i &= (\beta_{11})_i(\gamma_1)_i - (\beta_{21})_i(\gamma_2)_i - |\Sigma_{31}|_i d_2 \\
(f_2)_i &= (\alpha_{12})_i(\gamma_1)_i - (\alpha_{22})_i(\gamma_2)_i - |\Sigma_{32}|_i d_1 \\
(g_2)_i &= (\beta_{12})_2(\gamma_1)_i - (\beta_{22})_i(\gamma_2)_i - |\Sigma_{32}|_i d_2 \\
(f_3)_i &= (\alpha_{13})_i(\gamma_1)_i - (\alpha_{23})_i(\gamma_2)_i - |\Sigma_{33}|_i d_1 \\
(g_3)_i &= (\beta_{13})_i(\gamma_1)_i - (\beta_{23})_i(\gamma_2)_i - |\Sigma_{33}|_i d_2,
\end{aligned}$$

where $i = 1, 2$, and

$$\begin{aligned}
(\alpha_{11})_i &= (\sigma_{22})_i c_1 - (\sigma_{23})_i b_1 \\
(\beta_{11})_i &= (\sigma_{22})_i c_2 - (\sigma_{23})_i b_2 \\
(\alpha_{12})_i &= (\sigma_{21})_i c_1 - (\sigma_{23})_i a_1 \\
(\beta_{12})_i &= (\sigma_{21})_i c_2 - (\sigma_{23})_i a_2 \\
(\alpha_{13})_i &= (\sigma_{21})_i b_1 - (\sigma_{22})_i a_1 \\
(\beta_{13})_i &= (\sigma_{21})_i b_2 - (\sigma_{22})_i a_2 \\
(\alpha_{21})_i &= (\sigma_{12})_i c_1 - (\sigma_{13})_i b_1 \\
(\beta_{21})_i &= (\sigma_{12})_i c_2 - (\sigma_{13})_i b_2 \\
(\alpha_{22})_i &= (\sigma_{11})_i c_1 - (\sigma_{13})_i a_1 \\
(\beta_{22})_i &= (\sigma_{11})_i c_2 - (\sigma_{13})_i a_2 \\
(\alpha_{23})_i &= (\sigma_{11})_i b_1 - (\sigma_{12})_i a_1 \\
(\beta_{23})_i &= (\sigma_{11})_i b_2 - (\sigma_{12})_i a_2.
\end{aligned}$$

Then these are substituted to Eq.(14), Eq.(15) is obtained and coefficients are expressed as:

$$\begin{aligned}
k_4 &= e_0^2 + e_3^2 + e_6^2 - D^2 e_9^2 \\
k_3 &= 2(e_0e_1 + e_3e_4 + e_6e_7 - D^2 e_9 e_{10}) \\
k_2 &= e_1^2 + 2e_0e_2 + e_4^2 + 2e_3e_5 \\
&\quad + e_7^2 + 2e_6e_8 - D^2 e_{10}^2 - 2D^2 e_9 e_{11} \\
k_1 &= 2(e_1e_2 + e_4e_5 + e_7e_8 - D^2 e_{10}e_{11}) \\
k_0 &= e_2^2 + e_5^2 + e_8^2 - D^2 e_{11}^2,
\end{aligned}$$

where,

$$
\begin{aligned}
e_0 &= (f_1)_2(\alpha)_1 - (f_1)_1(\alpha)_2 \\
e_1 &= (f_1)_2(\alpha)_1 + (g_1)_2(\alpha)_1 - (f_1)_1(\beta)_2 - (g_1)_1(\alpha)_2 \\
e_2 &= (g_1)_2(\beta)_1 - (g_1)_1(\beta)_2 \\
e_3 &= (f_2)_2(\alpha)_1 - (f_2)_1(\alpha)_2 \\
e_4 &= (f_2)_2(\beta)_1 + (g_2)_2(\alpha)_1 - (f_2)_1(\beta)_2 - (g_2)_1(\alpha)_2 \\
e_5 &= (g_2)_2(\beta)_1 - (g_2)_1(\beta)_2 \\
e_6 &= (f_3)_2(\alpha)_1 - (f_3)_1(\alpha)_2 \\
e_7 &= (f_3)_2(\beta)_1 + (g_3)_2(\alpha)_1 - (f_3)_1(\beta)_2 - (g_3)_1(\alpha)_2 \\
e_8 &= (g_3)_2(\beta)_1 - (g_3)_1(\beta)_2 \\
e_9 &= (\alpha)_2(\alpha)_1 \\
e_{10} &= (\alpha)_2(\beta)_1 + (\beta)_2(\alpha)_1 \\
e_{11} &= (\beta)_2(\beta)_1.
\end{aligned}
$$

B. Coefficients derivation of quartic equation in Section 4.2.2

In Eq.(17), if we put $\boldsymbol{s}_i = \boldsymbol{n}_1 \times \boldsymbol{n}_{bi}$ and $\boldsymbol{q}_i = \boldsymbol{n}_2 \times \boldsymbol{n}_{bi}$, we have the following expression:

$$\boldsymbol{d}_i = \mu \boldsymbol{s}_i + \boldsymbol{q}_i, (i = 1, 2)$$

Thus, Eq.(19) is obtained by substituting this to Eq.(18), and coefficients are expressed as:

$$
\begin{aligned}
k'_4 &= (\boldsymbol{s}_1, \boldsymbol{s}_2)^2 - (\boldsymbol{s}_1, \boldsymbol{s}_1)(\boldsymbol{s}_2, \boldsymbol{s}_2)\cos^2\theta \\
k'_3 &= 2\{(\boldsymbol{s}_1, \boldsymbol{s}_2)(\boldsymbol{s}_1, \boldsymbol{q}_2) + (\boldsymbol{s}_1, \boldsymbol{s}_2)(\boldsymbol{s}_2, \boldsymbol{q}_1) \\
&\quad - (\boldsymbol{s}_1, \boldsymbol{s}_1)(\boldsymbol{s}_2, \boldsymbol{q}_2)\cos^2\theta - (\boldsymbol{s}_1, \boldsymbol{q}_1)(\boldsymbol{s}_2, \boldsymbol{q}_2)\cos^2\theta\} \\
k'_2 &= \{(\boldsymbol{s}_1, \boldsymbol{q}_2) + (\boldsymbol{s}_2, \boldsymbol{q}_1)\}^2 + 2(\boldsymbol{s}_1, \boldsymbol{s}_2)(\boldsymbol{q}_1, \boldsymbol{q}_2) \\
&\quad - (\boldsymbol{s}_1, \boldsymbol{s}_1)(\boldsymbol{q}_2, \boldsymbol{q}_2)\cos^2\theta - (\boldsymbol{q}_1, \boldsymbol{q}_1)(\boldsymbol{s}_2, \boldsymbol{s}_2)\cos^2\theta \\
&\quad - 4(\boldsymbol{s}_1, \boldsymbol{q}_1)(\boldsymbol{s}_2, \boldsymbol{q}_2)\cos^2\theta \\
k'_1 &= 2\{(\boldsymbol{s}_1, \boldsymbol{q}_2)(\boldsymbol{q}_1, \boldsymbol{q}_2) + (\boldsymbol{q}_1, \boldsymbol{s}_2)(\boldsymbol{q}_1, \boldsymbol{q}_2) \\
&\quad - (\boldsymbol{s}_1, \boldsymbol{q}_1)(\boldsymbol{q}_2, \boldsymbol{q}_2)\cos^2\theta - (\boldsymbol{q}_1, \boldsymbol{q}_1)(\boldsymbol{s}_2, \boldsymbol{s}_2)\cos^2\theta\} \\
k'_0 &= (\boldsymbol{q}_1, \boldsymbol{q}_2)^2 - (\boldsymbol{q}_1, \boldsymbol{q}_1)(\boldsymbol{q}_2, \boldsymbol{q}_2)\cos^2\theta.
\end{aligned}
$$

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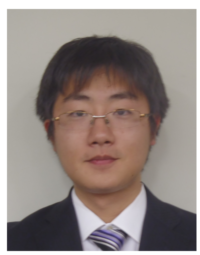

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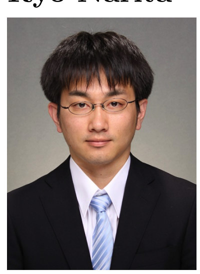

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